

# A Hilbert Space setting for $s \geq 1$ interactions which replaces Gauge Theory

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## Abstract

The recently discovered Hilbert space description of renormalizable interactions of higher spin  $s \geq 1$  fields requires to replace the pointlocal  $s=1$  vectorpotentials of indefinite metric (Krein space) BRST gauge theory by their stringlike counterpart in Hilbert space. It is shown that the Hilbert space positivity leads to new properties outside the conceptual range of the gauge theoretic description: topological aspects of Wilson loops, induced normalization terms, in particular Mexican hat type potentials for massive vectormesons coupled to Hermitian scalar field and a possible role of string-localization in confinement and "darkness".

## 1 Introduction

There is no property of quantum theory which is more important than the positivity coming from its operator formulation in Hilbert space. Born's probability interpretation of quantum theory (QT) depends on it, and in quantum field theory (QFT) it is a direct consequence (without invoking Born definition) of modular localization theory together with the fact that all physical (i.e. finite energy) states are KMS states after restricting them to modular localized subalgebras. Hence in QFT there is a direct relation of quantum causal localization in Hilbert space with statistical ensemble probability (the ensemble of observables localized in a spacetime region) with which Einstein had no problems [1] [2].

Yet gauge theory starts from covariant *pointlike* vectorpotentials which are incompatible with a Hilbert space and rather act in a suitably defined indefinite metric Krein space; it is not the linear structure of Hilbert space but rather the for quantum theory indispensable nonlinear positivity aspects which is violated in gauge theory and only partially recovered in terms of gauge invariance. Whereas for free fields it is trivial to recover the associated pointlike

field strengths whose application to the vacuum state generate a Hilbert space, in the presence of interactions the indefinite metric becomes deeply enmeshed with the matter fields (which in zero order are free fields in Hilbert space), so that one needs a rather elaborate operator gauge formalism which requires an extension of the Krein space setting by "ghost operators") in order to be able to extract a subset of local observable fields which act in a Hilbert space. In this gauge formalism important physical states (e.g. charged states) remain outside the formalism.

Gauge theory describes the vacuum sector i.e. the Hilbert subspace generated by the gauge invariant observables acting on the vacuum but the gauge-variant field (which includes the charge-carrying matter fields of QED) have no physical meaning. The BRST gauge formalism is a consistent combinatorial perturbative formalism which is (apart from the mentioned vacuum sector) outside the functional analytic operator control of quantum theory. It is somewhat of a miracle that the extension of the BRST  $s$ -invariance to global objects as scattering process leads to results which are not only consistent with the formalism but also pass many observational tests. In the words of Raymond Stora, the main protagonist of BRST gauge theory "gauge theory is a miracle and one does not understand why, as far as we know it, the formal rules seem to cover observational results of particle physics". Most physicists from the older generation (including myself) knew that quantum gauge theory is a successful "placeholder" for a still unknown perturbative renormalization theory for  $s \geq 1$ . Stanley Mandelstam and Bryce DeWitt attempted to go beyond gauge theory at a time when important concepts were still missing.

*The lack of Hilbert space positivity is a gaping wound in the QFT description of  $s \geq 1$  interactions and in particular of  $s = 1$  gauge theory. It is the principle aim of the present paper to overcome this limitations and obtain new results and interpretations which gauge theory misses, as correct description of the charge screening properties of interacting massive vector mesons in particular in couplings with Hermitian fields (the Higgs model).*

This is very different from the situation in classical gauge theory where pointlike vectorpotentials have a well-defined conceptual status as classical fields and gauge transformations transform classical vectorpotentials into others in such a way that the observable field strengths remain invariant. Vectorpotentials are useful classical objects even though they do not explicitly appear in Maxwell's equations. In contrast to QFT, Hilbert space positivity and probability have no conceptual counterpart in classical theory, and hence it is not surprising that the alleged quantization parallelism between classical and quantum field theories on which Lagrangian quantization is based fails precisely for  $s \geq 1$  QFTs as the result of a clash of the pointlike nature of the field description with the Hilbert space positivity of quantum theory. This problem is not limited to massless  $s \geq 1$  potentials where it is reflected in the impossibility to obtain pointlike covariant vectorpotentials in the  $s = 1$  from the covariantization of Wigner's unitary positive energy representations of the Poincaré group.

It also shows up in the nonrenormalizability of pointlike higher spin interactions; with other words behind pointlike nonrenormalizability of  $s \geq 1$  interac-

tions there hides a weakening of localization in such a way that the interaction becomes renormalizable if one works with stringlocal fields. The aforementioned gauge formalism is a formal trick to uphold pointlike localization at the expense of sacrificing Hilbert space positivity which at the end of the calculations. So quantum gauge theory comes at a high price in that the gauge invariant part only covers a small part of the full QFT although, as mentioned before, the application of the gauge formalism outside the vacuum sector leads to unmerited and (quoting Stora) not understood successes. It turns out that the renormalizable stringlocal matter fields can be used to define extremely singular pointlocal fields whose correlation functions are unbounded in momentum space (infinite  $d_{sd}$  in  $x$ -space); however for massless vectormesons only the stringlocal matter fields survive; in that case the strings are rigid [19] and different directions are not unitarily equivalent (Lorentz covariance is spontaneously broken [20]) which manifests itself in terms of the perturbative logarithmic infrared divergencies.

This limitation affects QED, where the absence of a pointlocal physical electron operator requires to substitute missing spacetime derivations of collision processes by momentum space prescriptions in terms of photon-inclusive cross sections for the scattering of charged particles. In order to improve the conceptual understanding of QFT there are two alternatives: either construct the charged sectors via representation theoretical concepts from the vacuum sector, or find a Hilbert space alternative to the Krein space gauge theory. The first strategy is successful for QFT with mass gaps<sup>1</sup>, but presents difficult and largely unsolved problems in case of the (gauge-invariant) vacuum sector in zero mass gauge theories [6]. The second strategy is to realize that the clash between localization and the Hilbert space structure can also be resolved by ceding on the side of localization; this causes no conceptual problems since pointlike localization in a Krein space outside a Hilbert space setting is anyhow a fake (if used outside the vacuum sector).

It turns out that the conceptual price is rather small compared with what one may have expected considering the fact that Hilbert space positivity is a very strong restriction. It simply consists in allowing fields localized on semi-infinite spacelike strings (in addition to those representing pointlike observables). However this requires the elaboration of a new formalism of renormalized perturbation theory which in case of  $s = 1$  replaces the gauge setting. The basic ideas and their application to second order perturbation theory, as well as their relevance for the future development of the Standard Model, is the main theme of the present note (see also [7], [2]). A systematic study of the problems encountered in the generalization of the Epstein-Glaser causal approach is in preparation [10] [11] whereas some preparatory remarks about the new setting can already be found in [12] [13] [14] [7].

The recognition of the conceptual origin of the problem is not new. Already Wigner in his 1939 representation theory [8] of  $m = 0$  finite helicity representations of the Poincaré group knew that covariant vectorpotentials which

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<sup>1</sup>The DHR reconstruction of the superselected sectors [4] and their amalgamation into a field algebra with an inner symmetry [5] for which the fields are not necessarily pointlike but could be semi-infinite stringlike.

are associated to  $(1/2, 1/2)$  representations of the Lorentz-group do not occur in the list of possibilities which arises from the covariantization of the unique ( $m = 0, |h| = 1$ ) Wigner representation. This permits an immediate generalization to  $s > 1$  tensorpotentials; there is no such problem with interaction-free pointlike massive potentials with short distance dimension  $d = s + 1$ ; they do not allow to take massless pointlike limits (the Proca potential,...) although this problem disappears by passing from potentials to field strengths. Zero mass limits however do exist after converting pointlike potentials into their covariant stringlike siblings which are associated with the same Wigner representation as the field strengths; there is no other way for maintaining the Hilbert space positivity and the standard relation with pointlike field strengths. All these observations on interaction-free potentials are well-known.

In the case of couplings involving *massive*  $s \geq 1$  fields, the problem between pointlike fields and the Hilbert space positivity is more subtle; in this case *the Hilbert space structure clashes with renormalizability*. The remedy is to convert the pointlike interaction into its stringlike analog and show that the renormalized perturbation theory involving covariant fields localized on semi-infinite spacelike strings is well-defined. The reason why this could work is that stringlocal potentials have  $d = 1$  independent of spin, which permits to construct renormalizable couplings in the sense of power counting ( $d_{int} \leq 4$ ) for any spin<sup>2</sup> in such a way that the local observables will remain pointlike (possibly pointlike composites of stringlocal fields). The constructions in this note will be limited to  $s = 1$ ; this is not only because in that case the new formalism has its simplest realization, but also since it permits confrontations of new results with observational physics; indeed it is the first contact of ideas coming from local quantum physics (LQP is the algebraic approach to QFT [4]) with observational properties of the Standard Model with many new theoretical aspects and new ways of explaining experimental observations.

Besides providing a basis for the extension of proofs of structural theorems (spin&statistics, TCP, the LSZ scattering theory from mass gaps,...) for which the Hilbert space positivity of quantum theory is essential and which therefore is not possible in a gauge setting, the existence of physical matter fields and massive Y-M gluons in the new formulation provides for the first time a QFT setting for *how confinement can be related to the infrared divergences of physical fields* in the massless limit. In all those structural properties, the Hilbert space positivity of the operator-algebraic setting (not taken care of in functional settings) plays an essential role, which explains why, apart from the vacuum sector, they are outside the range of gauge theory. Gauge theory is basically a perturbative combinatorial structure to which operator methods which rely on positivity (spectral representations, functional analysis, operator algebraic methods) cannot be applied.

In the existing literature one finds the observation that the so-called axial gauge is consistent with Hilbert space positivity. But what prevented to use this observation as a start of a Hilbert space formulation of  $s = 1$  interactions

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<sup>2</sup>In the present note the spin is assumed to be integer so the potentials are bosonic.

is the fact that only the interpretation of this unit vector  $e$  as a fluctuating spacelike direction of a field  $A_\mu(x, e)$  localized on the spacelike line  $x + \mathbb{R}_+ e$ ,  $e^2 \equiv e^\mu e_\mu = -1$  leads to a consistent formulation. In this new setting every field has its independent fluctuating  $e$ -variable, just as the fluctuating  $x$  which marks the start of a spacelike half-line. This requires in particular that a Lorentz transformation covariantly changes  $e$  (which contradicts the gauge-parameter interpretation). The non-covariant axial gauge setting finally fell into disgrace since it leads to confusing entangled ultraviolet-infrared divergence problems for which no solution was found. It turns out that it is precisely the directional fluctuation property in  $e$  which reduces the short distance dimension  $d = 2$  of the pointlike Proca field  $A_\mu^P$  to  $d = 1$  of the stringlocal  $A_\mu(x, e)$  and thus render the interaction fit for renormalization since it lowers the short distance dimension of e.g. massive pointlike QED from  $d_{int}^P = 5$  to the power-counting compatible value  $d_{int}^S = 4$  for the stringlocal interaction.

In fact the stringlocal construction of the S-matrix suggests a round-about definition of higher order pointlike interaction densities whose short distance behavior is precisely that expected from the unlimited increase of the momentum space polynomial degree; but in contrast to the direct pointlike setting, which leads to an ever increasing (with perturbative order) number of coupling parameters, the coupling strengths are those of the interaction-defining first order. Although the off-shell correlations of pointlike fields are expected to have short distance dimensions which increase with the perturbative order, this is not the case for the on-shell scattering amplitudes, as will be explained in these notes. In order to avoid conceptual confusions it is important to point out that the string-localization refers to fields and not to particles. Particles remain Wigner particles, and with the exception of the noncompact localized continuous spin Wigner representation spaces all particle spaces remain compact localizable.

The new Hilbert space setting has interesting consequences for the Standard Model. On the one hand, as already mentioned before, the Hilbert space nature of stringlocal massive Y-M fields (massive gluons) instead of pointlike unphysical Y-M fields in Krein space opens the possibility of an understanding of confinement in the limit of vanishing vectormeson mass. By applying resummation techniques to leading infrared logs<sup>3</sup> there are good reasons to expect, that by using the vectormeson mass as a natural covariant infrared regulator parameter, the  $m \rightarrow 0$  limit vanishes for all correlations which contain besides an arbitrary number of pointlike composites also a stringlike gluon (or "e-unbridged"  $q - \bar{q}$  pairs); this is the only known interpretation of the meaning of confinement which explains the non-observability of gluons/quarks and at the same time is consistent with the foundational causal localization principle (in a Hilbert space setting!) of QFT. On the other hand it also turns the "Higgs mechanism" from its head to its feet by realizing that the physical content of the Higgs model is nothing else than the renormalizable interaction of *massive vectormesons coupled to Hermitian* (instead of charged) *fields*  $H$ . In the correct description

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<sup>3</sup>Similar to the proof [17] of the vanishing of scattering amplitudes of charged particles in the presence of only a finite number of photons (in terms of perturbative resummation techniques).

there is no symmetry-breaking and *the Mexican hat potential is not put in, but is rather induced* in second order from the renormalizable stringlocal reformulation of a pointlike  $gA^PA^PH$  interaction; hence it is not surprising that the numerical coefficients of the induced potential depend on the ratio of the masses of the two fields. There is no place for couplings of massive vectormesons for symmetry-breaking (which symmetry?<sup>4</sup>) by nonvanishing one-point functions of gauge-variant fields; the conceptual difference between an induced Mexican hat potential and one put into the interaction in order to support the incorrect idea of mass creation through spontaneous symmetry breaking in the massless two-parametric scalar QED cannot be bridged by arguments which are consistent with QED and also not with the principles of QFT where masses of the model-defining elementary fields belong to the input data and only boundstate masses of particles interpolated by composite fields are predictions of the model.

The Hermitian model shares with its complex counterpart ("massive QED") the screening of its Maxwell charge. Since there is no particle/antiparticle counting charge, the characteristic property of the Hermitian coupling is the screened charge of the Maxwell-current which is the only conserved current of the abelian Higgs model. A conserved current of a spontaneous broken symmetry leads to a divergent charge (this is really the intrinsic definition of a spontaneously broken symmetry) whose large distance divergence is caused by the presence of a massless Goldstone boson (this is the content of Goldstone's theorem).

The stringlocal interaction associated to the pointlike  $gA^PA^PH$  brings a stringlocal selfadjoint scalar  $\phi(x, e)$  into the game which (unlike the Higgs field  $H$ ) shares its degrees of freedom with those of the massive vectorpotentials i.e. does not result from an additional coupling; the degrees of freedom are not changed by the presence of the "intrinsic escort" field  $\phi$ . The appearance of these stringlocal intrinsic escorts is a new phenomenon which results from the implementation of Hilbert space positivity in terms of stringlocal fields for  $s \geq 1$  interactions and has no counterpart in pointlocal models.

The recent proposal for a Hilbert space based formulation is not the first attempt to avoid the Krein space setting of  $s = 1$  gauge theory. Already in the 60s DeWitt [15] and Mandelstam [16] explored the possibility of circumventing gauge theory by implementing interactions directly in terms of Hilbert space-compatible field strengths. But without the awareness of the short distance singularity-reducing role of directional fluctuations of stringlocal vectorpotentials, a renormalizable theory in Hilbert space cannot be formulated. More important for the present Hilbert space based setting was the existence of a powerful structural theorem for QFTs with compact localizable (pointlike generated) observable subalgebras and a positive energy representation of the Poincaré group for which the energy momentum spectrum contains a mass gap [18]. Whereas the validity of scattering theory in the presence of a mass gap is hardly surprising, the assertion that the generating algebras of the full theory can be localized in (arbitrarily narrow) spacelike cones is somewhat unexpected.

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<sup>4</sup>Local gauge symmetry is not a symmetry but a formalism which permits to extract physics from a Krein space description (although it is in itself unphysical it leads to a physical Hilbert space subalgebra).

The previous observations about possible clashes between the pointlike localization of tensorpotentials and the Hilbert space positivity, as well as their resolution by working instead with stringlocal fields, find their natural explanation in this theorem. For zero mass vectormesons one cannot rely on this theorem; in that case the use of the quantum Gauss theorem [19] leads to the stringlocal nature of Maxwell-charge-carrying fields [19]. Those charged strings are *rigid*, in particular their direction cannot be changed by Lorentz transformations (spontaneous breakdown of Lorentz symmetry [20])

The re-translations of these findings from the algebraic "local quantum physics" (LQP) setting [4] into that of (operator algebra-) generating fields<sup>5</sup> (operator-valued distributions) means that in any QFT with a mass gap, which contains a pointlike generated observable subalgebra which generates the vacuum sector, the superselected charged sectors can be described in terms of stringlike covariant fields  $\Psi(x, e)$ . In this terminology "pointlike" is the special case of stringlike namely  $e$ -independence. To generate QFTs in terms of fields one does in particular not need fields which are localized on hypersurfaces ("branelike"-fields). The theorem does however not say anything about whether a particular concrete model can be generated by pointlike fields or if stringlocal fields are necessary. Here our perturbative results connect the necessity for using stringlike localization with the breakdown of renormalizability for their pointlike counterparts. In particular  $s \geq 1$  interactions have an interaction density with short distance scale dimension  $d_{int} > 4$  require the use of stringlike localization. The connection between nonrenormalizability and weakening of pointlike localization is, according to my best knowledge, a new result which will have consequences on earlier attempts to relate nonrenormalizability of certain models with modified renormalization group behavior [9]. The interesting question of an analog of the Callan-Symanzik equations for stringlocal interactions will remain outside the scope of the present paper. Gauge theories and the stringlike Hilbert space formulation as well as their relation to each other is the main topic of this note.

The next section entitled "kinematical prerequisites" presents the construction of the massive stringlike interaction-free vectorpotential  $A_\mu(x, e)$  in terms of their pointlike Proca siblings  $A_\mu(x)$  and a scalar escort field  $\phi(x, e)$  which are all members of the same stringlocal localization class. Whereas the massive Wilson loop in terms of the stringlocal field is equal to that in terms of the Proca field and hence its  $e$ -independence is manifest, its zero mass limit is  $e$ -independent in a topologically more subtle way. This explains the breakdown of Haag duality for multiply connected spacetime regions with interesting relations to the quantum mechanical Aharonov-Bohm effect; in fact this duality breakdown is a generic property of all *massless*  $s \geq 1$  potentials [21] [22]. The use of the indefinite metric pointlike potentials leads to wrong results, in other words the gauge description already breaks down if one considers global gauge invariant objects as Wilson loops; it remains strictly limited to pointlike generated gauge

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<sup>5</sup>Whereas the core of (causally closed compact) double cones are points, that of (causally closed noncompact) spacelike cones are covariant semi-infinite spacelike linear strings.

invariant local observables.

In the third section the kinematical preparation is used for the calculation of the string-independent second order S-matrix of massive vectormesons interacting with matter (massive scalar QED and the coupling to Hermitian  $H$ -fields<sup>6</sup>). For the  $H$ -coupling the "Mexican hat" potential, which has been imposed for the implementation of the alleged symmetry breaking "Higgs mechanism", emerges instead as a second order "induced potential" of the renormalizable coupling of a massive stringlocal vectormeson to  $H$  fields; *it is not the breaking of gauge symmetry but rather its upholding* which in massive vectormeson- $H$  couplings induces *a second order Mexican hat potential*. In the new stringlocal Hilbert space setting this arises from the string-independence of the second order S-matrix.

The concluding remarks present a resumé as well as an outlook. Here additional remarks about consequences of the stringlocal  $s \geq 1$  in Hilbert space (SLF) can be found and the stringlocal scenario for the expected confinement in the zero mass limit of self-interacting massive vectormesons is presented.

## 2 Kinematical prerequisites

With an  $s=1$  pointlike Proca field

$$A_\mu^P(x) = \frac{1}{(2\pi)^{3/2}} \int e^{ipx} \sum_{s_3} u_\mu(p, s_3) a^*(p, s_3) + h.c. \quad (1)$$

$$\langle A_\mu^P(x) A_\nu^P(x') \rangle = \frac{1}{(2\pi)^3} \int e^{-ip(x-x')} M_{\mu\nu}(p) \frac{d^3p}{2p_0}, \quad M_{\mu\nu}(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$$

one can connect two stringlocal fields

$$A_\mu(x, e) = \int_0^\infty F_{\mu\nu}(x + se) e^\nu ds, \quad F_{\mu\nu}(x) = \partial_\mu A_\nu^P(x) - \partial_\nu A_\mu^P(x) \quad (2)$$

$$\phi(x, e) = \int_0^\infty A_\mu^P(x + se) e^\mu ds$$

which are linearly related

$$A_\mu(x, e) = A_\mu^P(x) + \partial_\mu \phi(x, e) \quad (3)$$

This relation may also be directly derived in terms of the  $u$ -intertwiners of the three fields<sup>7</sup> defined in (2) for the computation of the intertwiners of the fields defined in (2). Since in the algebraic LQP setting fields which are relatively local (i.e. members of the same stringlocal Borchers class [23]) with respect to each other are considered as different "field-coordinatizations" of the same model (the same physics), this zero order relation is the kinematical prerequisite for

<sup>6</sup>The letter  $H$  stands for both: Hermitian and Higgs.

<sup>7</sup>I thank Jens Mund for pointing out that these relations correspond to linear relation between stringlocal intertwiners which have been introduced in [12]



its continued validity for the interacting potentials (its implementation is part of the renormalization process). For the perturbative Bogoliubov S-matrix only the free relation will be needed.

*At this point one begins to understand why the early attempts of Mandelstam and DeWitt failed. What was missing were two interrelated properties, namely the short distance scale dimension-reducing directional fluctuations of stringlocal potentials and the perception that the Hilbert space positivity requires the presence of a scalar "escort"  $\phi$  of the vectorpotential. This escort field does not generate new degrees of freedom; together with its "mother potential"  $A_\mu(x, e)$  it is simply the result of lowering the  $d_{sd} = 2$  of the Proca potential  $A^P(x)$  in order to obtain an interaction density which is below the power-counting limit  $d_{int} \leq 4$ . We will see that the escort enters the interaction density in an essential way.*

A useful reading of this relation (equivalent to the picture of lowering of  $d^P = 2$  to  $d^S = 1$  by one unit of  $d$  going into the  $e$ -fluctuations) is to say that the derivative of the  $d = 1$  scalar stringlocal "escort field"  $\phi$  compensates the leading short distance singularity at the price of weakening the localization from point- to stringlike. Each vectormeson potential has its  $\phi$ -companion which shares the degrees of freedom and the mass. It turns out that this mechanism permits a generalization to arbitrary high integer spin in which case there appear  $s$  stringlocal escort  $\phi$  fields with spins between 0 and  $s - 1$ ; the scalar  $\phi$  enters with  $s$  derivatives and the tensor indices of the different  $\phi$  together with the number of derivative in front always add up to  $s$ . In all cases the relation breaks down in the massless limit since there are neither pointlike Proca fields nor stringlocal massless  $\phi$ 's within the mentioned spin range; in this limit only the stringlocal  $A_\mu$  or equivalently the linear combination (3) survives-

Our main interest is  $s = 1$ . In this case the integration of the equation along a closed spacelike circle leads to

$$\begin{aligned} \oint A_\mu(x, e) dx^\mu &= \oint A_\mu^P(x) dx^\mu, \quad m > 0 \\ \oint A_\mu(x, e) dx^\mu &= \oint A_\mu(x, e') dx^\mu, \quad \forall e, e', \quad m = 0 \end{aligned} \tag{4}$$

since the integral along a spatial path of a gradient of  $\phi$  is the difference between the endpoint and the initial point which vanishes for coinciding points. The resulting independence of the Wilson loop operator from the direction  $e$  corresponds to the expected gauge invariance of the loop. In the zero mass limit, the difference of two line integral over the same curve but with  $e$ 's pointing into different directions is simply the difference between the  $\phi(x, e)$  and  $\phi(x, e')$ . Although the individual  $\phi$  are infrared divergent, their difference at the same point (the initial and final point of the Wilson loop) but with different directions stays infrared finite; hence the difference between two Wilson loops over stringlocal potentials with different string direction vanishes (the second line in 4). In this case there remains a topological imprint which the string dependence leaves behind.

The result is the breakdown of Haag duality for multiply connected spacetime regions. In such a case the algebra of a causally closed region  $\mathcal{A}(\mathcal{O})$ ,  $\mathcal{O} = \mathcal{O}'$  is not the same as the commutant of the algebra  $\mathcal{A}(\mathcal{O}')$  rather one finds

$$\mathcal{A}(\mathcal{O}) \subsetneq \mathcal{A}(\mathcal{O}')' \quad (5)$$

for  $\mathcal{O}$  a spacelike torus. The "thickened" Wilson loop is an operator in the right hand algebra which is not in  $\mathcal{A}(\mathcal{O})$ . The physical picture is that there are operators localized in a spacelike separated intertwining torus which have to penetrate the cylinder subtended from the loop into the  $e$ -direction somewhere whatever direction of  $e$  one chooses. The bigger the helicity, the higher is the genus up to which new violations of Haag duality occur.

This effect can also be directly derived from the equal time commutation relations of the field strengths without the use of their stringlike tensorpotentials (for  $s = 1$  this was shown in the unfortunately unpublished work [24]). The important point here is that the proof in terms of stringlocal potentials is simpler whereas the use of the Stokes theorem for pointlike potentials in Krein space is misleading. This is the simplest illustration for the necessity of a Hilbert space setting for potentials and the ensuing SLF setting. It should be clear that the stringlike localization is not the result of a playful fancy of particle theorists who set out to "try something else" and look for its potential physical use later on (which led to String Theory [1]), but rather the consequence of maintaining the Hilbert space positivity of quantum theory also for  $s \geq 1$ .

To avoid misunderstandings about the aim of the present work, it should be said that the central issue of this paper is not the change of the viewpoint about the nature of the QFT analog of the Aharonov-Bohm effect in the Hilbert space setting (instead of the usual gauge-theoretic pointlike setting in Krein space) but rather the changes which Hilbert space positivity causes in the ongoing research of the Standard model (in particular the Higgs issue and confinement). The simplicity of the above presentation of interaction-free spacetime loops is meant as a pedagogic illustration that even free field properties of *global* gauge invariants come out incorrect; the physical range of gauge theory is strictly limited to gauge invariant local observables.

Thinking of the functorial relation between Wigner's positive energy representation spaces for the Poincaré group and interaction-free quantum fields and the associated local nets of operator algebras, it is interesting to note that the operator algebraic breakdown of Haag duality for multiply connected spacetime regions has a *spatial counterpart in modular localized toroidal subspaces of the Wigner representation space*. The spatial counterpart of the stringlocal vectormeson field is the covariant stringlocal Wigner wave functions which together with its opposite frequency part defines a hyperbolically propagating classical wave functions for a classical stringlocal vectorpotential. Different from the standard use of pointlike classical vectorpotentials, the stringlocal vectorpotential extends to spacelike infinity and thus prevents the formation of compact Cauchy data for potentials.

The Wilson loops formed with the correct classical potentials, although being causally separated from a magnetic flux inside the Wilson ring, still "feel"

its presence because the flux lines have to penetrate the walls of one of the infinite spatial cylinders which are associated to the different choices of the  $e$ 's; this is a topological imprint which the stringlocal vectorpotential leaves behind if one forms a classical Wilson loop which looses geometric memory of the  $e$  of its vectorpotential. This is the way in which the presence of the magnetic flux is perceived despite the geometric causal separation between the potential in the loop and the magnetic flux inside. The Aharonov-Bohm effect is a quasiclassical relic (quantum mechanical matter in a classical vectorpotential) of this breakdown of Haag duality and admits higher helicity generalization ( $m = 0, |h|$  finite); it looses its exotic appeal by abandoning the idea of pointlike potentials. The observance of this discrepancy between the naive geometric picture of apparent spacelike separation and the classical limit of the more hidden stringlocal nature of quantum potentials removes all feelings of nonlocal magical aspects of the A-B effect.

This provides strong support for the Hilbert space formulation of  $s \geq 1$  fields as compared to the gauge theoretic setting. It shows that the often as magic perceived mismatch between the naive geometric view and that coming from causal localization of the A-B effect disappears if one permits the more fundamental QFT the chance to tell its classical counterpart to use the correct description (classical stringlocal fields and their topological implication for loops). This philosophy is opposite to that of "quantization" where the (Euler-Lagrange) description is used as a classical crutch to enter the world of QT. Stringlocal fields in Hilbert space are not solutions of Euler-Lagrange equations and the consistency of gauge theory with QT is limited to the vacuum sector (and even there it appears more a matter of luck than of exigency).

It is also interesting to note that the better known "Coulomb gauge" is a Hilbert space description which results from the covariant stringlocal potential by directional averaging within a spatial hyperplane

$$A_\mu^C(x) = \int A_\mu(x, \vec{e}) \frac{d\Omega}{4\pi}, \quad A_0^C(x) = -\frac{1}{4\pi} \int d^3y \frac{\text{div } \vec{E}(\vec{y}, t)}{|\vec{y} - \vec{x}|}, \quad \vec{A}^C(x) = \frac{1}{4\pi} \int d^3y \frac{\text{rot } \vec{H}(\vec{y}, t)}{|\vec{y} - \vec{x}|} \quad (6)$$

This exposes its covariance property in terms of the action of the Lorentz group on the time-like vector orthogonal to the hypersurface.

In massive  $s \geq 1$  theories the clash between Hilbert space and pointlike localization and its resolution through the use of stringlike tensorpotentials is reflected in the fact that behind pointlike nonrenormalizability there looms a weakening of localization; the attempt of a pointlike description leads to singular matter fields with short distance dimension  $d = \infty$  (unlimited increase with the perturbative order). Mathematically the formal pointlike fields are singular to such an extent that a smearing with all compact supported spacetime testfunctions is not possible. The Wightman localization property can only be recovered in terms of renormalizable stringlocal fields  $\Psi(x, e)$  which can be smeared with all Schwartz testfunctions  $f(x, e)$  with compact supports in  $\mathfrak{D}(\mathbb{R}^4)$  tensored with 3-dim. *de Sitter*), see next section.

Linear relations between high dimensional pointlike fields and their lower

dimensional stringlike siblings (which for  $s=1$  reduce to (3)) are the key for the conversion of nonrenormalizable pointlike interaction densities into affiliated renormalizable stringlocal interactions. Here are two illustrations:

- Scalar massive QED with the first order interaction density (products of operators are always Wick-ordered products) which is to be multiplied by a numerical coupling strength

$$\begin{aligned} L^P &= A_\mu^P j^\mu, \quad j^\mu = i\varphi^* \overleftrightarrow{\partial}^\mu \varphi \\ L^P &= L - \partial^\mu V_\mu, \quad L = A_\mu j^\mu, \quad V_\mu = \phi j_\mu \end{aligned} \quad (7)$$

here we used (3). Whereas the  $L^P$  has operator short distance dimension  $d = 5$ , which is too high in order to be within the renormalizable power counting range of  $d \leq 4$ , the stringlocal action has  $d = 4$ . The incriminated dimension 5 has been absorbed into a derivative term where it can be disposed of in the adiabatic limit; in this way the first order S-matrix of the pointlike  $L^P$  is the same as that of the stringlike  $L$ . In fact one does not have to know the polynomial expression, rather its form together with that of  $V_\mu$  results from the requirement that  $L - \partial V$  is independent of  $e$ . We may say that, given the field content, the renormalizable first order interaction is *self-induced and (in the cases tested up to now) unique*. This induction principle extends to all orders (see next section).

- The coupling of a massive vectormeson to a Hermitian field is

$$\begin{aligned} L^P &= A^P \cdot A^P H = L - \partial^\mu V_\mu \\ L &= A \cdot (A^P H + \phi \partial H) - \frac{m_H^2}{2} \phi^2 H, \quad V_\mu = A_\mu^P \phi H + \frac{1}{2} \phi^2 \partial^\mu H \end{aligned} \quad (8)$$

up to renormalizable  $H^3, H^4$  self-interactions whose coupling strengths turn out to be fixed by higher order induction so that at the end there is just one first order coupling  $g$  and the two masses of the interaction-defining fields. The latter are usually not counted but for comparison with the Higgs mechanism it is helpful to mention them.

In both cases the leading short distance singularity is "peeled off" from the pointlike scalar  $L^P$  in terms of a divergence of a vector in analogy to the peeling off in (3) by the gradient of a scalar. The  $d = 5$  of the pointlike interaction density has been converted into the power-counting-compatible  $d = 4$  of the stringlike density. The split into a stringlocal  $d = 4$  interaction and a divergence is unique up to divergence-free additional contributions to  $V_\mu$ .

### 3 Higher order string independence of the S-matrix and induced Mexican hat potentials

If the construction of higher order interaction densities  $TL(x_1, e_1) \dots L(x_n, e_n)$  would not involve the time-ordering of operator-valued distributions, the deriva-

tives in  $\partial^\mu V_\mu$  could be taken outside the time-ordering and the previous relations (78) would have a straightforward  $n^{th}$  order extension. The singularities at point- and string-crossings prevent this, and in case of interactions between pointlike fields Epstein and Glaser [25] established rules for the inductive construction of time ordered products based on the perturbative implementation of causality with minimal scaling degree (which is not larger than the naive high energy divergence degree of corresponding Feynman integrals). Those couplings for which the minimal scaling degree stay finite independent of the perturbative order are called renormalizable. They are those couplings for which the scaling degrees remains finite and the pointlike fields remain localizable. A simple criterion for renormalizability is  $d_L \leq 4$  where  $d_L$  is the short distance scaling degree of  $L$ ; the fulfillment of this power-counting criterion the prerequisite for renormalizability. What renders nonrenormalizable theories physically worthless (apart from possible phenomenological use) is not so much the unbounded increase for  $p \rightarrow \infty$  but primarily the growth of the number of coupling parameters associated with an ever increasing number of counterterms.

An systematic extension of the E-G method to string-crossings has not yet been published [10], but fortunately such systematics is not yet needed if, as in the present work, one's aim is to direct attention to the *derivation of new concepts and results* from explicit second order model calculations. The main new message is that requirement of string independence of the S-matrix places new restrictions on renormalizations which lead to the concept of *induced normalization terms* i.e. their couplings are uniquely determined in terms of the basic model-defining first order coupling and the masses and spins of the interaction-defining free fields. This is similar to the gauge theoretic setting where the second order  $A_\mu A^\mu$  term of scalar QED is induced from the first order coupling by gauge symmetry. The fundamental difference is however that in the new setting one does not have to invoke a new symmetry, rather the induction is the consequence of causal localization in a Hilbert space setting (modular localization). The main point here is not that the induced counterterm structure is more or less isomorphic<sup>8</sup> to that of the gauge setting, but rather that the Hilbert space positivity has additional physical consequences about the structure of global string-independent quantities which the gauge setting cannot reproduce; the simplest free field illustration is the loss of the above topological property in the used of Wilson loops.

In second order the  $e$ -independence is more subtle since the direct definition of the pointlike  $TL(x)^P L^P(x')$  fails as a result of nonrenormalizability. However the requirement

$$d_e(TLL' - \partial^\mu TV_\mu L') = 0, \quad (9)$$

where the bracket is considered as a zero differential form in  $e$  (on the 3-dim. unit de Sitter space) on which a differential form operator  $d_e$  acts, is perfectly meaningful since the short distance degrees of  $LL'$  and  $V_\mu L'$  does not exceed 8.

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<sup>8</sup>Most interesting are however additional terms in the SLF setting which have no counterpart in the BST gauge formalism.

A more symmetric form which simultaneously takes care of  $e, e'$  is

$$\begin{aligned} d(TLL' - \partial^\mu TV_\mu L' - \partial^{\mu'} TLV'_\mu + \partial^\mu \partial^{\nu'T} V_\mu V'_\nu) &= 0, \quad d = d_e + d_{e'} \quad (10) \\ T(LL')^P &:= TLL' - \partial^\mu V_\mu L' - \partial^{\mu'} LV'_\mu + \partial^\mu \partial^{\nu'} V_\mu V'_\nu \end{aligned}$$

Although a direct perturbative treatment of the pointlike second order interaction density would conflict with renormalizability, the indirect definition in terms of the string-independent bracket as in the second line is perfectly reasonable. It is again the encoding of the highest scale dimensions into derivatives of lower dimensional operators within the power counting limit ( $d = 4n$  in  $n^{th}$  order) which permits to define pointlike products whose direct  $n^{th}$  order scale dimension  $d = 4n + n$  would exceed the power-counting limit and, if treated in the usual pointlike setting, would lead to an ever increasing number of coupling parameters since the induction mechanism would be absent. The relations (9,10) have straightforward  $n^{th}$  order extensions, but here their perturbative model implementation will be limited to second order.

The similarity of (3) with a gauge transformation suggest that the connection of the stringlocal matter field and its strongly singular (not Wightman-like, pointlike nonrenormalizable) pointlike sibling should be

$$\psi(x) = e^{-ig\phi(x,e)}\psi(x,e) \quad (11)$$

However the conceptual content of these relations is quite different than that of gauge transformations; they do not stand for a symmetry transformation ("gauge symmetry") but rather relate two "field-coordinatizations" in Hilbert space which belong to the same localization class (relative locality with respect to each other). The proof of these off-shell relations in the presence of interactions requires an extension of the Stückelberg-Bogoliubov-Epstein-Glaser (SBEG) on-shell formalism. In fact the representation for the S-matrix is a special case of that for fields and their correlation functions; all these formulas require to take the adiabatic limit<sup>9</sup>.

Fortunately in massive QED this is not needed since the matter field only enters the interaction in form of the conserved current and the SBEG formalism for the on-shell S-matrix only deals with time-ordered products of free. The expected off-shell connections between renormalizable stringlike fields and their formally nonrenormalizable pointlike siblings contains however the interesting message that the kind of nonrenormalizability addressed in the present work is a result of the clash of enforced pointlike localization with the Hilbert space positivity; a clash which can be removed by passing to the renormalizable stringlocal formulation.

The application of (9) to the model of massive scalar QED (7) requires to expand the time ordered product into Wick-products. The component without contractions fulfills this relation trivially as a consequence of  $d_e(L - \partial^\mu V_\mu) =$

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<sup>9</sup>The SBEG field construction is opposite to that of LSZ scattering theory; in the latter case one starts from the field correlations and computes the S-matrix as the "cream on the cake".

$d_e L^P = 0$ . For the 1-contraction component this is not the case since the anomaly  $\mathfrak{A}$

$$\begin{aligned} A & : = d_e(T_0 L L' - \partial^\mu T_0 V_\mu L')|_{1-contr.} = -d_e N + \partial^\mu N_\mu \\ A' & = A(x, e \longleftrightarrow x', e'), \quad \mathfrak{A} = A + A' \end{aligned} \quad (12)$$

where  $T_0$  refers to the "kinematical" time ordering for which all derivatives act outside the  $T_0$  e.g.

$$\begin{aligned} \langle T_0 \partial_\mu \varphi(x) \partial'_v \varphi^*(x') \rangle &= \partial_\mu \partial'_v \langle T_0 \varphi(x) \varphi^*(x') \rangle \\ \langle T_0 \partial_\mu \varphi(x) \partial'_v \varphi^*(x') \rangle & : = \langle T_0 \partial_\mu \varphi(x) \partial'_v \varphi^*(x') \rangle + i g_{\mu\nu} c \delta(x - x') \end{aligned} \quad (13)$$

According to the Epstein-Glaser normalization rules the scaling degree 4 (logarithmically diverging) propagator of derivative of scalar fields permit a scaling degree 4 preserving renormalization in terms of a delta term with a yet undetermined parameter  $c$ . The contributions to the anomaly coming from the action of the divergence  $\partial_\mu$  on contractions in  $T_0 j_\mu(x) j_\nu(x')|_{1-contr.}$  are

$$\partial^\mu \langle T_0 \partial_\mu \varphi(x) \varphi^*(x') \rangle = (\partial^\mu \partial_\mu + m^2) \langle T_0 \varphi(x) \varphi^*(x') \rangle - m^2 \langle T_0 \varphi(x) \varphi^*(x') \rangle = -i \delta(x - x') - reg. \quad (14)$$

It is precisely these numerical anomalies of time-ordered propagators which determine the above operator anomalies. The result of the contraction combinatorics is

$$N = \varphi^* \varphi A \cdot A', \quad N_\mu = \delta \varphi^* \varphi \phi A'_\mu \quad (15)$$

where the  $A'_\mu$  stands for  $A_\mu(x, e')$  and  $\delta$  is  $\delta(x - x')$ . We also use the fact that  $d_e \partial_\mu \phi = d_e A_\mu$ . For the renormalization of  $T_0 L L'$  the  $N_\mu$  (which corresponds to the renormalization of  $T_0 V_\mu L'$ ) is not important. Hence the relevant  $N$  part of the full anomaly is symmetric in  $e$  and  $e'$

$$\varphi^* \varphi (A \cdot A' + A' \cdot A) \quad (16)$$

The reader immediately recognizes that this induced contact term which must be added to  $T_0 L L'$  corresponds to the quadratic term in the gauge theoretic formulation of scalar QED. In the present setting it is simply the consequence of the string-independence of the S-matrix. The philosophy underlying the present setting suggests to absorb this term into a re-definition (renormalization) of the time ordering by

$$\begin{aligned} T &= T_0 \text{ for propagators of scaling degree } d < 4 \\ T &\text{ as in (13) with } c = -1 \text{ for } d = 4 \end{aligned}$$

In this case the  $T_0$  contraction of the propagator of the derivative of the fields coming from the anomaly can be absorbed in the redefinition (13)  $T_0 \rightarrow T$  of the contraction in  $T_0 L L'$  so that the  $e$ -independence (9) is fulfilled with the  $T$  ordering instead of the  $T_0$ . One can then directly check that (10) really defines a second order pointlike interaction density without having to introduce

a counterterm with a new coupling (as it would be necessary in the pointlike formalism in Krein space before imposing the gauge invariance condition). In fact the formula (10) guaranties the independence of the scattering amplitude on the string directions.

The string-independence in the Hilbert space setting  $d_e S = 0$  corresponds to  $sS = 0$  with a nilpotent  $s$  in the BRST gauge setting, where the definition of  $s$  requires to enlarge the already unphysical Krein space in terms of ghost operators. The computation is analogous, except that there are no  $A'_\mu$ . The Hilbert space positivity for massive QED leads to on-shell results for  $2 \rightarrow 2$  scattering which have the same formal appearance as those coming from gauge theory, even though the concepts and calculations are different. In the following we sketch the operator gauge derivation a la Scharf; this should also be seen as a recompensation of the early results of the University of Zürich group [27] which were left on the wayside by the Standard Model caravan and eventually succumbed to the maelstrom of time<sup>10</sup>.

The self-induced  $e$ -independent  $d_e(L - \partial V) = 0$   $L, V$  pair of a trilinear  $A - H$  coupling (8) has its BRST gauge-theoretic counterpart in  $s(L^K - \partial V^K) = 0$  where the fields now act in the Wigner-Fock Krein space. Apart from the fact that in addition to  $u^K = s\phi^K$  we also have an anti-ghost field<sup>11</sup>  $\tilde{u}$  and the first order induction of a trilinear interaction leads to (we surprise the superscript  $K$  for the individual fields) and differences in the normalization of the  $\phi^K$  field, the formal expression for the first order gauge interaction is analog to (8)

$$L^K = m \left( A \cdot AH - H \overset{\leftrightarrow}{\partial} \phi \cdot A - \frac{m_H^2}{2m^2} H \phi^2 + bH^3 + u\tilde{u}H \right) \quad (17)$$

$$Q_\mu^K = m(uA_\mu H - \frac{1}{m} u \phi \overset{\leftrightarrow}{\partial}_\mu H) = sV_\mu^K$$

$$sA = \partial u, \quad sH = 0, \quad s\phi = u, \quad s\tilde{u} = -(\partial A + m^2 \phi) \simeq 0 \quad (18)$$

Up to the  $H$  self-interactions these trilinear terms are the unique (up to  $V_\mu$  contributions with vanishing divergence) renormalizable induced first order terms. The third line is the action of  $s$  on the individual free fields which act in the Krein space analog of the Fock space. The second order anomalies  $sT_0 LL' - \partial^\mu T_0 Q_\mu L'$  and the corresponding term with  $Q \rightarrow L, L' \rightarrow Q'$  (prime denotes  $x \rightarrow x'$ ) lead to the additional (induced) contribution [26]

<sup>10</sup>We (Jens Mund and myself) only became aware of this contribution after our work on stringlocal fields [12] [13]. Here these results have been rewritten into our formalism in order to facilitate comparisons.

<sup>11</sup>The ghost field corresponds to  $u := d_e \phi$ , but the anti-ghost has no counterpart since  $s\tilde{u} = (\partial A^K + m^2 \phi^K)$  and the corresponding SLF expression vanishes since it is an operator relation in Hilbert space.



$$T_0 L^K L^{K'} + i\delta(x - x')(A \cdot AH^2 + A \cdot A\phi^2) - i\delta(x - x')R_{scharf} \quad (19)$$

$$R_{Scharf} = -\frac{m_H^2}{2m^2}(\phi^2 + H^2)^2, \quad V_{Scharf} \equiv g^2 R + \text{first order } H, \phi - \text{terms}$$

$$V_{Scharf} = g^2 \frac{m_H^2}{8m^2}(H^2 + \phi^2 + \frac{2m}{g}\phi)^2 - \frac{m_H^2}{2}H^2 \quad (20)$$

The remaining step consists in absorbing the quadratic in  $A$  contributions into a redefinition of the time-ordered product  $T_0 \rightarrow T$  (as in the previous case of scalar massive QED) and to verify that the absence of third order anomalies requires to introduce a  $cH^4$  self-coupling and fixes the parameters  $b$  and  $c$ .

The net result is the  $H$ - $\phi$  local potential  $R$  of degree 4 (20). The appearance of  $g^{-1}$  terms results from writing the potential into the symmetry-breaking form of the Higgs mechanism from where one may read off the Mexican hat parametrization of the Higgs mechanism (the quartic self-coupling of scalar QED and the shift in field space). It shows that that the latter is incompatible with the logic of renormalized perturbation theory, whereas the mechanism of induction potential is its logical second order result. The lesson to be learned is that, whereas at the frontiers of the foundational and sometimes highly speculative research as particle theory one sometimes is led to use metaphors as placeholders for incompletely understood issues, the problem starts when the placeholder nature is not recognized in due time. Admittedly this was not easy since the coupling of Hermitian fields to massive vectormesons which vanish in the massless limit is somewhat unaccustomed for physicists coming from QED.

For the details of the anomaly calculations and the derivation of the (up to second order) induced potential (??) we refer to Scharf (formula 4.1.38 [26]). As in the case of the Schwinger-Swieca screening of the Maxwell current of interacting massive vectormesons, we recommend to the readers to take their time to look up the cited historical papers in order to convince themselves how close some individuals came to the correct understanding of interactions involving massive vectormesons.

This should have caused the ringing of bells, but the Standard Model Theory community was already in the grip of Big Science; it was too late for a perception and a critical discussion of these findings; more recent attempts to remind the particle theory community of some of these forgotten (or never noticed) results remained without avail [28]. What was still missing in order to realize that the unfortunate broken symmetry picture is inconsistent with the computed result was to notice that the Schwinger-Swieca screening [32] of the Maxwell charge of interacting massive vectormesons (the only conserved current in the  $H$ -model) is completely different from the charge associated to the conserved current of a spontaneously broken symmetry. A deeper sociological analysis of these strange occurrences in the heart of particle theory will be left to historians and archeologists of science.

The new SLF Hilbert space formalism does not only confirm these pre-

electronic insights<sup>12</sup> but also adds an interesting twist to it; the Hilbert space positivity leads to the presence of additional induced contributions to the Mexican hat potentials which however vanish on the diagonal  $e = e'$ . The definition of pointlike interaction densities 10 within the Hilbert space setting also reveals a new property which has no counterpart in the gauge setting. These pointlike densities are special cases of the polynomial expression which one would obtain in second order direct pointlike nonrenormalizable expression, except that instead of new counterterm coupling parameters the latter are now not independent but rather fixed in terms of the first order couplings and masses. As a consequence the on-shell polynomial degree is lower than its off-shell counterpart; with other words the scattering amplitudes have a much better high-energy behavior than that of pointlike correlation functions. This is the momentum space counterpart of "peeling off" high degree derivative terms and dispose them in the adiabatic limit, a mechanism which is difficult to keep track of in momentum space and which shows that the consequences of the Hilbert space positivity for  $s \geq 1$  (which requires the use of stringlike localization) cannot be encoded into Feynman graphs.

These deviations of the Hilbert space setting from gauge theory are interesting, but their detailed derivation go beyond the task set for the present paper and will be addressed in separate work [29]. As mentioned in the introduction, the physical credibility of the gauge setting is restricted to the gauge invariant vacuum sector of local observables which strictly speaking does not include the global S-matrix. The frequent formal similarity between the gauge setting and the Hilbert space formulation may be interpreted as an unexpected extension of the physical limitations of gauge theory.

In passing it is interesting to see that the screening of the identically conserved Maxwell current can already be seen in zero order of the Maxwell current

$$\begin{aligned} j_\mu^M &: = \partial^\nu F_{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (21) \\ &\text{in zero order} \quad \partial^\nu F_{\mu\nu} \simeq m^2 A_\mu, \quad \partial^\mu A_\mu = 0 \end{aligned}$$

The corresponding zero order Maxwell charge vanishes

$$\text{observation : } Q^M = \int j_0^M(x) d^3x, \text{ in zero order } j_0^M(x) \simeq A_\mu^P(x), \quad \int A_0^P(x) d^3x = 0 \quad (22)$$

*Thm : The Maxwell charge of a massive vectormeson is always screened*

Whereas for couplings to Hermitian fields this is the only conserved current, the complex matter fields of massive QED also admit another conserved current whose conserved charge counts the number of charges minus anti-charges. The charge screening is lost and both currents coalesce in the massless limit.

It was Schwinger [31] who conjectured this property of massive vectormesons which was later proven as a structural (nonperturbative) theorem by Swieca [32] [33]. In this context it may be interesting to mention that the conserved

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<sup>12</sup>Setting  $e = e'$  in the scattering amplitudes, the results formally agree (up to differences in the  $\phi^K$  versus  $\phi$  normalizations).

current in the case of a spontaneously broken symmetry leads to a divergent charge whose large distance divergence is due to the presence of a zero mass Goldstone boson. Interestingly the structural proof of the Goldstone theorem (which has no relation with nonvanishing one-point functions of fields<sup>13</sup>) is also due to Swieca [34] [35], in fact he emphasized that the large-distance diverging charge is the definition of the meaning of spontaneous symmetry breaking. He probably had the most profound knowledge of both phenomena and he tried to attract attention away from the incorrect Higgs symmetry breaking by using in his publication the terminology "Schwinger-Higgs screening" in his publications. But unfortunately he was unable to stem the growing tide in favor of symmetry breaking. This and the 40 year reign of the Higgs spontaneous breaking mechanism instead says a lot about the present state of post Standard Model.

In retrospect it appears very conspicuous that the same arguments and equations which started from massless scalar QED and postulated a symmetry breaking (gauge symmetry?) by performing a "field shift" of a gauge variant field were almost simultaneously independently presented by at least 3 other authors/groups of authors in addition to Peter Higgs. For fairness it should be mentioned that during the first years after these proposals there were also publications which pointed out that gauge symmetry is not a physical symmetry but rather a method to extract local observables from a Krein space setting and as such it is not fit for being broken. Note that the above induced Mexican hat potential can also be obtained by imposing BRST gauge invariance on a massive vectormeson- $H$  interactions [26]; Hence it is its imposition of gauge invariance in interactions of massive  $A_\mu$  with Hermitian fields and not its breaking in massless scalar QED which is consistent with the foundational principles of QFT.

These remarks should not be understood as coming from someone whose research was motivated by the desire to revolutionize particle theory. To the contrary it is the increasing worry that a particle theory which lost its contact with its own pre-electronic past has moved into a blind alley which motivated the author to write this article. An important observation in the aftermath of the discovery of the Standard Model as the "Schwinger-Higgs-Swieca screening mechanism" of massive vectormesons should never have been allowed to be lost in the maelstrom of time. More conceptual and historical observations which receive their support from the ongoing impact of the new SLF Hilbert space setting can be found in [1]. The many updates of this article since 2011 reflect the continuous development of these ideas.

The biggest gain of insight from the new  $s \geq 1$  Hilbert space setting is expected in the area of self-interactions between massive vectormeson (Y-M couplings). One expects of a setting whose principle task is to classify renormalizable interactions within the Hilbert space setting of renormalizable  $d_e$ -induced first order interaction densities that it achieves something equivalent to what in classical gauge field theory is obtained with the help of the mathematical fibre-bundle setting. For self-interacting massive vectormesons of *equal masses*

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<sup>13</sup>The shift in field space is a mnemonic trick to produce a first order interaction with such a divergent charge associated to a conserved Goldstone current, but it has no intrinsic physical meaning.

the couplings should be restricted to just one coupling strength. In particular for three mutually interacting massive gluons of equal mass the result should look like

$$\begin{aligned}
L &= \sum \varepsilon_{abc} (F^{a,\mu\nu} B_\mu^b A_\nu^c + m^2 B^{a,\mu} A_\mu^b \phi^c), \quad V^\mu = \sum \varepsilon_{abc} F^{a,\nu\mu} (A_\mu^b + B_\nu^b) \\
d_e(L - \partial^\mu V_\mu) &= 0. \text{ Ansatz } L = \text{sum of 4 terms in } A, \phi \\
&\sum f_{abc}^1 F^{a,\mu\nu} A_\mu^b A_\nu^c, \quad \sum f_{abc}^2 A^{a,\mu} A_\mu^b \phi^c, \quad \sum f_{abc}^3 A^{a,\mu} \partial_\mu \phi^b \phi^c, \quad \sum f_{abc}^4 \phi^a \phi^b \phi^c
\end{aligned}$$

where for notational convenience we used the notation  $B = A^P$  for the pointlike Proca potentials. The last line denotes the 4 structures which can contribute to  $L$ . The requirement that  $L - \partial V$  is  $e$ -independent (pointlike) is very restrictive and leads to the first line where the  $V_\mu$  is determined up to a term with vanishing divergence<sup>14</sup>. If one defines the pointlike  $L^P$  as the content of the bracket than the first order pointlike S-matrix is identical to its stringlike counterpart or equivalently: the two  $L$ 's are adiabatically equivalent (the boundary term from the divergence of  $V_\mu$  vanishes in the adiabatic limit)

$$\int L^P = \int L, \quad L^P \stackrel{AE}{\simeq} L \quad (25)$$

This is the beginning of an extremely restrictive *induction mechanism* which has no counterpart in the nonrenormalizable pointlike  $s \geq 1$  setting. For the full Lie-algebra structure one has to proceed to the induced second order [30].

These observations generalize those which were already made in the abelian case namely the locality principle together with Hilbert space positivity leads to restrictions between couplings which correspond to those of classical gauge theory (the geometry of fibre bundles). Here they are simply the result of the Hilbert space positivity which for interactions which couple  $s \geq 1$  fields requires the use of string-localization. There is absolutely no need for any support from the fibre-bundle setting of classical gauge theory; QFT does not need any "crutches" from classical field theory such as those provided by the classical-quantal parallelism of quantization. Any quantum fields obtained from covariantizing Wigner's classification of positive energy representation of  $\mathcal{P}$  can be coupled to a scalar density which defines the first order interaction density of a QFT and in case its short distance dimension falls within the power-counting range  $d_{sd} \leq 4$  the interaction density is on the best way to define a renormalizable model of QFT. The above "self-induction" mechanism also works for unequal masses; in this case the  $f$ 's depend also on mass-ratios.

The full calculation up to second order will be contained in [30]. Our main point here is that the specification of the prescribed *field content together with the  $d_e$  induction requirement and the renormalizability restriction  $d_{int} \leq 4$  from power-counting determines the interaction  $L$  and the (in the S-matrix adiabatic limit disappearing) divergence contribution of  $V_\mu$  (again up to  $\partial \tilde{V} = 0$  terms).*

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<sup>14</sup>Similar conclusions follow from the imposition of  $s$ -invariance of the S-matrix [26].

*Possibly missing restrictions are expected to come from second order.* This means in particular that one does not have the freedom of classical field theory to add interactions; the field content, the power-counting restriction together with the induction mechanism fix the form of the  $s \geq 1$  quantum interaction. In particular the second order contributions required by classical gauge invariance cannot be imposed; rather the induction property, behind which hides the powerful Hilbert space positivity, governs the form of the quantum interaction.

This is particularly interesting for  $s = 2$  interactions where the massive stringlocal potentials  $g_{\mu\nu}(x, e)$  associated to their pointlike "Proca" siblings  $g_{\mu\nu}^P(x)$  and two escort fields  $\phi$  and  $\phi_\mu$  already enter the lowest order interaction density; this rigidity of the induction remains in the massless limit and stands in an interesting contrast with the Lagrangian quantization of the classical Einstein-Hilbert action from which "quantum gravity" could profit. In this connection it is important to point out that renormalization group properties (analogs of the Callen-Symanzik equations) can only be expected to hold for the stringlocal Wightman fields (and not for the singular pointlike fields). Even if the asymptotic freedom property would not have been based on a consistency property but rather on the beta function of an established Callan-Symanzik equation of massive (infrared-finite) QCD in the pointlike gauge setting, its physical relevance would still leave doubts. Only a derivation based on the SLF Hilbert space setting can definitely close this issue.

The formal  $m \rightarrow 0$  limit in the stringlocal interaction density is generally not sufficient for understanding the content of the massless limit. If there are off-shell (logarithmic) infrared divergences this should be taken as an indication of a radical change in the field-particle relation. Confinement manifests itself in the sense of vanishing of correlations for  $m \rightarrow 0$  which contain in addition to pointlike composite also stringlike gluons or quarks. This is expected to be obtained by resummation techniques of the leading  $m \rightarrow 0$  logarithmically divergent terms [1].

## 4 Résumé and outlook

The Hilbert space positivity for higher spin interactions leads to a weakening of localization and requires the replacement of nonrenormalizability of pointlike fields by stringlocal interaction densities. This replaces the BRST gauge theory in Krein space whose physical range of validity of gauge theory is limited to (pointlike generated) gauge invariant local observables; global gauge invariant operators as Wilson loops are already outside its range since they miss the topological origin of the breakdown of Haag Duality (for  $s = 1$  the Aharonov-Bohm effect).. In the SLF Hilbert space setting fields are generically stringlocal and physical; the subset of pointlike fields correspond to the gauge invariant local observables of gauge theory.

The SLF formalism leads to profound conceptual and computational changes. There are 3 basic classes of renormalizable couplings of massive vectormesons: couplings with complex matter, with Hermitian  $H$  matter and couplings among

themselves including a combination of these 3 classes of couplings; in all cases the masses of vectormesons and the matter fields (in accordance with the principles of QFT) belong to the the model defining field content in terms of which the first order interaction density is defined. The perturbative renormalization formalism secures that these input masses remain equal to the masses of the elementary particles of the model. Up to this point there is no difference to the  $s < 1$  pointlike renormalization theory. What is however new for  $s \geq 1$  is that in a Hilbert space setting renormalizability requires the vectormeson fields to be stringlocal and escorted by fields  $\phi$ , a property which through higher order perturbation also "infects" the matter fields. As a consequence the demand that certain physical global objects as the S-matrix stay string-independent ( $e$ -independent) leads to a new phenomenon called induction; instead of counterterms with free coupling parameters the coupling strengths of the induced terms are determined in terms of the interaction-defining first order coupling including the masses of the coupled fields. For the  $H$ -coupling the induced potential in  $H$  and the escort field  $\phi$  has the form of a Mexican hat. In the operator gauge setting<sup>15</sup> the result is (up to difference in normalization) the same but the SLF derivation of the Mexican potential from the BRST  $s$ -invariance of the S-matrix takes place outside the narrow limitation of gauge-invariant local operators and hence has less physical credibility.

The result vindicates the Schwinger-Swieca charge-screening mechanism behind the  $H$ -coupling of the Higgs model. Since Schwinger's screening conjecture referred to massive QED, the coupling of a Hermitian field which first appeared in the somewhat metaphoric veil of the Higgs model was by all means an enrichment of possibilities of couplings of massive vectormesons with matter. Apart from the omnipresent stringlocal scalar Hermitian escorts  $\phi$ , all couplings of massive vectormesons to matter (including the  $H$ -coupling) can be removed by applying Occam's razor and therefore are hardly of the foundational significance which the mass-creating Higgs mechanism attributes to them. The appearance of a massive gluonium state with the quantum numbers attributed to the Higgs particle in a system of self-interacting massive vectormesons would change the conceptual situation. In such cases of such bound state problems (e.g. hadrons in terms of quarks) one usually looks for phenomenological description and the  $H$ -coupling may well be such a description.

Significant progress from the new ideas is expected in the hitherto unsolved problems which hide behind the perturbative logarithmic  $m \rightarrow 0$  infrared divergencies of stringlocal fields. Here the Hilbert space positivity is expected to show its full strength; the induction mechanism for the lowest order self-interactions between massive vectormeson relates the possible as independent presumed couplings; for equal masses the trilinear first order couplings must be antisymmetric and the second order induction is expected to have a Lie-algebra structure. In other words the properties which hitherto entered via the classical geometry of fibre-bundles and quantization into the QFT of  $s = 1$  interactions

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<sup>15</sup>In the more common functional setting of perturbation theory (Feynman graphs) the induction mechanism is easily overlooked.

(and finally took the form of operator BRST gauge theory) can be obtained without classical crutches alone from the Hilbert space positivity of the SLF formulation of interactions between massive vectormesons.

There is another important attention-demanding aspect of the Hilbert space based perturbative stringlocal formulation of  $s \geq 1$  interactions. According to the LSZ reduction formula the scattering amplitudes are simply the on-shell restrictions of the momentum space correlation functions. Hence unless a special mechanism intervenes, one expects that the off-shell high energy degree is passed to the scattering amplitudes. The "peeling property", i.e. the disposal of the leading short distance singularities of the form of divergencies  $\partial V$  in the adiabatic limit is easily overlooked in momentum space, in particular since they cannot be encoded into Feynman graphs and already occur for tree-contributions to the scattering amplitude. The validity of graphical representations (Feynman graphs) is limited to pointlike renormalizable interactions for  $s < 1$ ; formally they also hold for the gauge setting in Krein space except that scattering amplitudes are already outside the range of gauge invariant local observables. This raises the important question whether the more popular functional formalism (instead of an operator formulation) remains reliable for  $s \geq 1$  interactions.

Looking back at the history of renormalized perturbation theory it is conspicuous that there has been no noteworthy conceptual investment with computational consequences since the post WWII discovery of renormalized perturbation theory in the wake of Lagrangian quantization. Even the later extensions of these techniques to nonabelian gauge theories did not require significant new conceptual investments; as before, prescriptions for the removal of infinities and consistency checks for the so-obtained finite expressions was all that was needed. Later conceptual progress which allowed to derive these results from the iterative perturbative implementation of the foundational causal localization principle was hardly noticed by the majority of practicing particle theorists. The "think as (or often after) you pull up your sleeves and compute along" attitude became the motto of "robust" and very successful particle theory to the extent that it led to extreme claims e.g. that any physically inspired mathematically correct calculation will find its material realization in one of the imagined parallel universes.

It seems that this successful period in particle theory had reached its apogee at the time of the discovery of the Higgs mechanism. Identical Lagrangian manipulations which led to identical conclusions were obtained in at least four independent publications at approximately the same time (a rather unique event in the history of particle theory), and while on the experimental side there has been a steady progress mostly confirming theoretical ideas, particle theory went through a already 40 year lasting period of impressive conceptual stagnation. The concepts and even their formulation (including the mass-giving "fattening" of photons) have remained identical, and completely reasonable ideas as Schwinger's charge screening for massive vectormesons and its mathematical derivation from first principles were allowed to vanish in the maelstrom of time.

This is the clearest indication that the conceptual reservoir of the post WWII particle theory has been used up and new investments are urgently needed. The

present proposal of extending the renormalizable Hilbert space operator setting of perturbation theory to higher spins  $s \geq 1$ , and confront the localization problems caused by maintaining the Hilbert space positivity head-on, is meant as a contribution to this goal.

Finally some remarks about the history behind the present ideas may be helpful. Although it may be seen as a new startup of old ideas of DeWitt [15] and Mandelstam [16], this is not the way in which it arose. The booster was rather our solution of the old problem of the field theoretic content of Wigner's infinite spin positive energy representation of the Poincaré group [12]. Although it was already clear by 1970 that this representation does not admit an associated pointlike field description [36], the first clue in what direction to look came from the application of *modular localization* to the positive energy Wigner representation [12]; the remaining problem of converting this observation into quantum fields associated to Wigner's infinite spin representation was solved in the cited work. Whereas in that case the string localization was endemic (every field, including composites has a noncompact localization), its use for finite helicity representation was only necessary if one were to use potentials instead of field strengths. The short distance scale dimension reducing role of string-localization was noted in [12] and the first remarks about how to use this in order to convert nonrenormalizable pointlike couplings into power-counting renormalizable interactions of stringlocal fields can be found in [13] and [14].

Meanwhile there exist strong indications that the noncompact localizable infinite spin Wigner "stuff" is more than a booster for directing the conceptual attention to string-localization. Its solely noncompact modular localization (no pointlike composites) leads to "inert matter"; accepting the standard picture about counter registration of particles, a particle event is interpreted as a reduction of a field state into a quasi-local counter-centered particle wave function [4]. In case of the inherently noncompact Wigner stuff the localization of fields and particles are both noncompact and the situation of a counter registration can not be realized. However the stability and the coupling to gravitation which are properties shared by all positive energy representations; for this reason the Wigner stuff is the ideal candidate of "darkness". In fact interacting confined matter and free noncompact Wigner stuff stand in a conceptually interesting contrast [37]. Although they share the *irreducibility* of their zero mass strings (which accounts for the darkness of free strings and the confinement of interacting strings) whereas the strings of electrically charged fields of QED remain reducible [7].

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